SOLUTION OF SHORT QUESTIONS

Short Questions

Write the short answers of the following Questions:

Q.1: Define the law of sines.

(IIA-2018), (IIA-2019), (IA-2021), (IIA-2021)

Sol. In any triangle ABC, with usual notations.

$$\boxed{\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}}$$

Q.2: Define the law of cosines.

(IA-2017), (IIA-2017), (IIA-2020), (IA-2021)

Sol. In any triangle ABC, with usual notations.

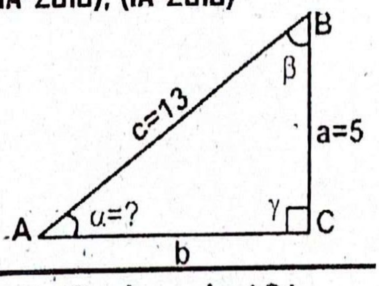
i. $a^2=b^2+c^2-2bc\cos\alpha$

ii. $b^2=c^2+a^2-2ca\cos\beta$

iii. $c^2=a^2+b^2-2ab\cos\gamma$

Q.3: In right triangle ABC, $\gamma=90°$, $a=5$, $c=13$, then find the value of angle 'α'. **(IIA-2016), (IA-2019)**

Sol. We know that, from figure:



$$\sin\alpha=\frac{a}{c}\Rightarrow\sin\alpha=\frac{5}{13}$$

$$\alpha=\sin^{-1}\left(\frac{5}{13}\right)\Rightarrow\boxed{\alpha=22°37'}$$

Q.4: Given that $\gamma=90°$, $\alpha=35°$, $a=5$, find angle 'β'.

Sol. We know that in any triangle: **(IA-2016), (IA-2018)**

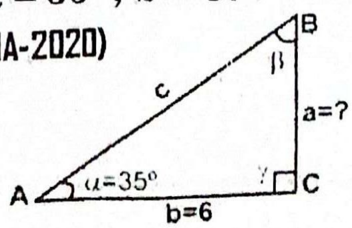
$$\alpha+\beta+\gamma=180°$$

$$\beta=180°-\alpha-\gamma$$

$$\beta=180°-35°-90°\Rightarrow\boxed{\beta=55°}$$

Q.5: In right triangle ABC, $\gamma=90°$, $\alpha=35°$, $b=6$. Find side 'a'. **(IA-2017), (IIA-2020)**

Sol. We know that, from figure:



SOLUTION OF SHORT QUESTIONS

$$\tan\alpha = \frac{a}{b} \Rightarrow \tan 35^\circ = \frac{a}{6}$$

$$6\tan 35^\circ = a \Rightarrow \boxed{a = 4.2}$$

Q.6: Given that: $\alpha = 30^\circ, \gamma = 135^\circ, c = 10$, **find 'a'.**

Sol. Using law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

we take : $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$$\frac{a}{\sin 30^\circ} = \frac{10}{\sin 135^\circ}$$

$$a = \frac{10\sin 30^\circ}{\sin 135^\circ}$$

$$a = \frac{5}{0.7071}$$

$$\boxed{a = 7.07}$$

Q.7: In any triangle ABC, if $a = 20, c = 32, \gamma = 70^\circ$ **find 'α'.**

(IIA-2016), (IIA-2018), (IIA-2020)

Sol. Using law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

we take : $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

$$\sin\alpha = \frac{a\sin\gamma}{c}$$

$$\sin\alpha = \frac{20\sin 70^\circ}{32}$$

$$\sin\alpha = 0.5873$$

$$\alpha = \sin^{-1}(0.5873)$$

$$\boxed{\alpha = 35^\circ 57' 58''}$$

Q.8: In any triangle ABC, if $a = 9, b = 5, \gamma = 32^\circ$, **find 'c'.** **(IA-2019)**

Sol. Using law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = (9)^2 + (5)^2 - 2(9)(5)\cos 32^\circ$$

$$c^2 = 29.67$$

$$\sqrt{c^2} = \sqrt{29.67} \Rightarrow \boxed{c = 5.45}$$

SOLUTION OF SHORT QUESTIONS

Q.9: The sides of a triangle are 16 , 20 & 33 meters respectively. Find its greatest angle.

(IA-2017), (IA-2019)

Sol. Let, $a = 16$, $b = 20$, $c = 33$

As side 'c' is greatest so, we will find angle 'γ'.

Using law of cosines : $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$2ab\cos\gamma = a^2 + b^2 - c^2$

$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab} = \frac{(16)^2 + (20)^2 - (33)^2}{2(16)(20)} = -0.6765$$

$$\gamma = \cos^{-1}(-0.6765) \Rightarrow \boxed{\gamma = 132^\circ 34'}$$

Q.10: Define angles of elevation and depression.

Ans. <u>Angle of elevation:</u> **(IIA-2016)**

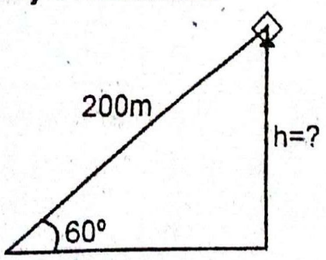
If the line of sight is upward from the horizontal, the angle is called angle of Elevation.

<u>Angle of depression:</u> **(IA-2016)**

If the line of sight is downward from the horizontal, the angle is called angle of Depression.

Q.11: A string of a flying kite is 200m long, and its angle of elevation is 60°. Find the height of the kite above the ground taking the string to be fully stretched.

(IA-2018)



Sol. In this figure: $\sin 60^\circ = \dfrac{h}{200}$

$200\sin 60^\circ = h$

$\boxed{h = 173.20\,m}$

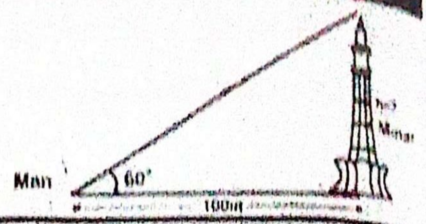
Q.12: A minaret stands on the horizontal ground. A man on the ground, 100m from the minaret, find the angle of elevation of the top of the minaret to be 60°. Find its height.

Sol. From figure, we know that: **(IA-2017), (IA-2019)**

SOLUTION OF SHORT QUESTIONS

$$\tan 60^\circ = \frac{h}{100}$$

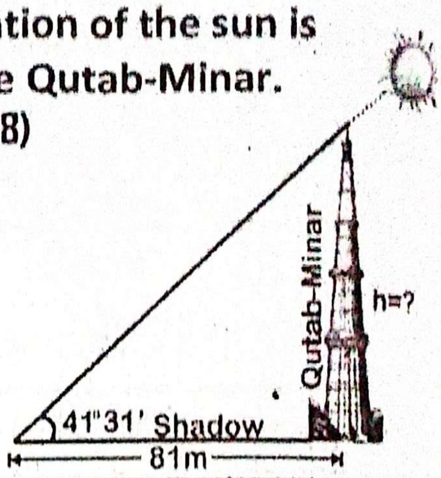
$$100\tan 60^\circ = h \Rightarrow \boxed{h = 173.20\text{m}}$$



Q.13: The shadow of Qutab-Minar is 81m long when the measure of the angle of elevation of the sun is $41^\circ 31'$. Find the height of the Qutab-Minar.

(IA-2016), (IIA-2018)

Sol. From figure, we know that:



$$\tan 41^\circ 31' = \frac{h}{81}$$

$$81\tan 41^\circ 31' = h$$

$$\boxed{h = 71.70\text{m}}$$

Q.14: In any triangle ABC in which:

$b = 45$, $c = 34$, $\alpha = 52^\circ$, find 'a'.

(IIA-2016), (IA-2017), (IA-2018), (IIA-2019), (IA-2022)

Sol. Using law of cosines : $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$$a^2 = (45)^2 + (34)^2 - 2(45)(34)\cos 52^\circ$$

$$a^2 = 2025 + 1156 - 1883.92$$

$$a^2 = 1297.07$$

$$\sqrt{a^2} = \sqrt{1297.07} \Rightarrow \boxed{a = 36.01}$$

Q.15: In any triangle ABC in which:

$a = 16$, $b = 17$, $\gamma = 25^\circ$, find 'c'.

(IA-2019), (IIA-2020)

Sol. Using law of cosines : $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$$c^2 = (16)^2 + (17)^2 - 2(16)(17)\cos 25^\circ$$

$$c^2 = 256 + 289 - 493.03$$

$$c^2 = 51.97 \Rightarrow \sqrt{c^2} = \sqrt{51.97} \Rightarrow \boxed{c = 7.2}$$

SOLUTION OF SHORT QUESTIONS

Q.16: In any triangle ABC in which:

$a = 5$, $c = 6$, $\alpha = 45°$, **find** '$\sin\gamma$'.

(IA-2016), (IIA-2019)

Sol. Using law of sines : $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

we take : $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\gamma}$

$5\sin\gamma = 6\sin 45°$

$\sin\gamma = \dfrac{6\sin 45°}{5} \quad \Rightarrow \quad \boxed{\sin\gamma = 0.8458}$

Q.17: In any triangle ABC, in which:

$b = 45$, $c = 34$, $\alpha = 52°$, **find** 'a'. **(IIA-2018)**

Sol. Using law of cosines : $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$a^2 = (25)^2 + (37)^2 - 2(25)(37)\cos 65°$

$a^2 = 625 + 1369 - 781.84$

$a^2 = 1212.16$

$\sqrt{a^2} = \sqrt{1212.16} \quad \Rightarrow \quad \boxed{a = 34.82}$

Q.18: In any triangle ABC, in which:

$a = 3$, $b = 7$, $\beta = 85°$, **find** 'α'.

(IIA-2017), (IA-2018)

Sol. Using law of sines : $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

we take : $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} \quad \Rightarrow \quad \dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b}$

$\sin\alpha = \dfrac{a\sin\beta}{b} = \dfrac{3\sin 85°}{7} = 0.4269$

$\alpha = \sin^{-1}(0.4269) \quad \Rightarrow \quad \boxed{\alpha = 25°16'}$
